INTER-, MULTI-, AND TRANSDISCIPLINARY MODELLING AND MODELS

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Abstract. *Perhaps the idea connected to the priority of object, method, theory or model in the modern sciences' way of thinking could have been the aim of this investigation, but inter-, multi-, and trans-dimensionality of the same model dominates the present, in science and education. Modern model is less (uni)disciplinary and dominantly inter-, multi-, and transdisciplinary. The majority of the lines in this article tries to identify an adequate answer to an ordinary investigation, everything being placed under the dazzling form of a simple question: What is the contemporary content and meaning of the word model and the real sense of the action of modelling in the modern science? The central part of this article develops three important aspects for maintaining the real development of the inter-, multi-, and trans-disciplinary modern science: i) the new paradigm of scientific model and the ascendant importance modelling in the scientific research and academic education; ii) the basic conditions of and modelling; iii) the specific architecture and paradoxes of the inter-, multi-, and transdisciplinary models and scientific modelling. Some final remarks underline the necessity of a better appreciation and implementation of modelling in education and research, and the reconfiguration of a remarkable future of the model in science.*

Keywords: *model, modelling, modelling paradigm, method, theory, scientific way of thinking, inter-, multi-, and transdisciplinarity. (uni)disciplinarity*

1. INTRODUCTION

Inter-, multi-, and transdisciplinarity have a seemingly common origin and delimit characteristic forms of the antonyms of unidisciplinarity as the knowledge acquired with the help of the unique discipline or unidisciplinarity. (Uni)disciplinarity, in its open sense, without the natural pretense of knowing everything in a limited domain is an initial natural stage of the limited scientific human knowledge and understanding. Multidisciplinarity presupposes simultaneity in the process of applying the thinking of several sciences (disciplines), interdisciplinarity designates the establishment of relations between several sciences and, finally, transdisciplinarity appears "*between disciplines, along them and above them*." [1]

(Uni)disciplinary modelling appears less and less in modern practice, respectively if we give the qualifier of (uni)disciplinary model to a model built on a dominant thinking of a discipline. The frequency of this (uni)disciplinary modelling has a minimum value as one can investigate based on a team of researchers, the real proof being its appearance in very rare case in modern scientific knowledge and research.

Multidisciplinarity of modelling presupposes that the study and research of an object of reality be realized from several points of view, descended from the multiplied thinking of several sciences at the same time. Both the modelling and the multidisciplinary research object, depending on the research result, will eventually become more enriched.

Interdisciplinarity of modelling has a diverse nuanced and purpose in direct relation to minimum two (uni)disciplinary visions, be it open, assuming phenomena, concepts and general modelling laws common to several disciplines that analyze in as varied contexts as possible, to highlight the multiple facets and possibilities of application of concepts, and laws in an increasingly varied disciplinary sphere. Interdisciplinarity favors the horizontal transfer of concepts, methods and models from one discipline to another. In interdisciplinarity, wone can detail three different degrees of their transfer, on neighboring fields, from other disciplines: i) applicative transfer; ii) epistemological transfer (cognitive); iii) transfer generating new disciplines [2] (e.g. transfer of statistical-mathematical methods in economics gave birth to econometrics, the first science created through methodological transfer, which later became a multidisciplinary type, the transfer of the econometric modelling in the space of the financial economy saturated with uncertainty generated by the theory of probabilities the financial econometric model). Interdisciplinarity of modelling is also a process of focusing or concentrating on the interstitial problems of several sciences or disciplines. The interweaving of disciplines and the coordination of research can end by adopting the same set of fundamental concepts or general methodical elements, ie by delimiting a new field of knowledge or a new discipline.

Transdisciplinarity of modelling is considered a superior form of interdisciplinarity that presupposes concepts, methods, methodology and a language that tend to become universal, being generated dynamically by the action of numerous stratifications of reality about reality etc.

The complex multidisciplinarity in modelling as a form of interpenetration of disciplines, consisting in joining certain elements of various disciplinary models, highlights their common aspects, and involves a symmetrical communication between various specialists in various disciplines, in their own axiometry.

Complex multidisciplinarity in modelling does not mean the simple juxtaposition or coexistence of models belonging to most disciplines in a single field, but it is accompanied by a transition through interdisciplinarity (e.g. a permanent transfer of informational and methodological models from discipline to discipline) to transdisciplinarity as modelling purpose, in the limiting sense of a broad dissolution of all sciences into only universal one and their models in a general and unique model, a complex fusion in a huge scientific universe or multiverse of contemporary sciences and scientific models.

Alfred Marshall inimitably described mental modelling as one that needs three great intellectual faculties: a) perception; b) motivation; c) imagination (above all). Imagination meaning is to intuit and connect the direction of events that are far away or under control. a perceptible surface, with causes and effects, which are located at a similar distance or below the same surface. [3]

Mental modelling is the representation of our deep understanding of a portion of reality that we have realized rather theoretically and less methodically and as an experimental consequence. Any mental modelling must be flexible in the sense of reconsidering the reality studied or synthesized as a field of information extended beyond the numerically limited universe or, in other words, beyond simple mathematical modelling, becoming a filter through which to interpret reality, to it is possible to act rationally on it and, especially to select based on an optimal prognosis, the most appropriate solution or variant of action for the situation. In a sense, everything that differentiates and consolidates the idea of logical, philosophical, mathematical, physical, economic, etc. thinking can be identified and redefined one by one through the specific concept of mental modelling. [2; 3]

There are general disadvantages, respectively of most mental modelling (from the comprehensive difficulty, to the subjectivity of their interpretation, from their imperfection as a methodology, to their incompleteness as a degree of coverage of reality, etc.), but also specific (as they seem to be the names given to the components of reality, with the meaning of symbolic words, as a tool for knowing the permanent and invariable essence of things in linguistic modelling or how minimalism and non-contradiction appear in logical modelling, etc.).

Becoming famous in the vast realm of thought, the problem of the circularity of formal systems finds that the desire to express knowledge in a formal way is illusory and that it exists in main formal logic systems or related systems, relatively simple assertions or theorems that cannot be solved. In that system, the respective assertions or theorems from the analyzed model are neither provable nor unprovable, like Gödel's famous problem [4].

Contrary to the mental model, the experimental model gives priority to the idea that the reality studied as a system or as a whole, represents more than the sum of the parts, the experiment continuously offering corrections to the aggregate reality, as a support for modelling. Experimental modelling characterizes physical thinking and is much closer to nature or reality.

The solution of physical models seeks to circumvent the problem of ambiguity or contradiction by continuous experimental rectification, and happily ensures the completeness of modelling by minimalism, by returning to nature or reality, in a continuous, non-speculative but interrogative way to validate the assumptions of physical knowledge. Although apparently not dominated by details, the physical model is much more capable than other specific scientific models of reconsidering their importance through the process of validating or invalidating hypotheses with the experimental thinking's help.

2. SCIENTIFIC MODELLING AND MODELS

Science by definition is open to change and indeed science as a whole is constantly changing. The primary scientific methodology is the same and has not really needed changing (also open to change): Observe, Theorize, Test Theory with data and evidence, adjust if needed, and then let it lie out there to be tested independently by others in the future and be adjusted if needed. Methodology meaning is how to find the truth through evidence, mathematical (in modern times especially statistical) and logical argument, finally through validation or invalidation old or new theories. René Descartes was advocating in his *Discours de la méthode* that a broad interdisciplinarity seems more possible in the science's future. "*Hence we must believe that all the sciences are so interconnected, that it is much easier to study them all together than to isolate one from all the others. If, therefore, anyone wishes to search out the truth of things in serious earnest, he ought not to select one special science; for all the sciences are conjoined with each other and interdependent*…" [5]

Science as knowledge, is derived from the Latin word *scientia*, and defines a systematic ensemble of knowledge connected with nature, society, education, research and thinking. "*Scientics or scientology currently represents the science of science, an investigation into the way in which the study of nature through observation and reasoning has evolved all through several millennia of human activity. Logic is, in its capacity as a "thought that thinks of itself" the first scientific discipline achieving almost unanimous recognition."* [6]

"Mathematics has come, as a result of the studies on quantities and hierarchies, turned into theorems by means of logical derivation, to be called a science of quasi-general usefulness, yet, without physics and its necessary limits and aspect of finiteness, introduced into mathematical reasoning, the results of scientific knowledge would rather be axiomatic systems of infiniteness. Through methodically measuring the manner in which the characteristics of populations vary statistics rounds up logics, mathematics and physics, while emphasizing the importance of observation and reasoning, in much the same way as physics does, by means of experiment and simulation, in its perpetual attempt to grasp reality. And so, the broad spectrum of natural science is reached, where science describes a systematic study, or the knowledge acquired subsequent to that study conducted on nature, starting from human nature (anatomy, sociology, etc.), up to animal, and even inanimate, nature (biology, geology, etc.)." [6]

Science emerges when at least four major elements are joined together: "a *characteristic part of reality, a method for investigation, an original theory and a special model for projection."* [6] All of these elements are somehow similar with *"air, earth, water and fire of the scientific thought, combining the dangers of the new connexion between reality and theory, with idealization and pragmatism, even sometime in an excessive manner."* [3]

Who could have constituted the beginnings: *"the method, the theory or the model of thinking in the process of investigation a special reality and defining a science and his status? The abundance of data has imposed the need of clarifying the importance of the mixture of method, theory and model in the contemporary science. The synthetic quantitative determinations have often been defined as methods and they hide in their large veil of* *indicators the real meanings of qualitative information, edifying for understanding the nature, structure, territoriality, and differentiated dynamics of the specific reality. The new theories trying to understand the causes, and effects of specific phenomena, and the new tendencies, the original temporal and spatial projections have invited and still invite to reflection. Using the same way in which the small models have created new sciences, we try to understand the birth and growth of the live science's way of thinking, and their new paradigms."* [1; 3] The modern science becomes a brief transformation of knowledge, from the most usual and simple access to information into a special complex way of thinking, teaching, learning and researching.

Why is the method so important? First, one can find an answer to Stefan Odobleja "*Neither the subject, nor the object are the determining factors for defining a science, but this could be only the specific method, which is indeed the essential factor generating its own paradigms*". [1;7] On the other hand, primarily nature of the reality's phenomenon reveals at least three dimensions: naturally devoid of finitude, that the first is the presence of unknown or of the limit afforded by the "observed object", the second is the limit of the observer's competence and especially the third is the limit of the method used in the characteristic analysis. Thus method is always a necessity and a limit of each science. Comprehensive knowledge of relativity or type of comprehensive analysis, limiting the presence untouchable result of "*unknown*" always gives other researchers the chance to try new solutions, because there is no specific limit in human way of thinking.

The limitation caused by observers or researchers means to understand the millennial tribute to the serenity of their exigency, and especially to reveal their own incompetence: "*I remember the days when scribes let the page empty seats*" are Confucius' words underlying the decency and modesty of any researcher or scientist… [3]

The science is also the analysis of a section of the reality as object, using methods inside a specific theory and model as an instrument for the future projections. The modern science means also a special theory able to match in a practical manner to a part of reality, and the essential instruments of forecasting and projection remain models. A scientific theory is "*a model of the universe, or a restricted part of it, and a set of rules that connect the magnitudes in the model to the observations that the researcher makes*" in the usual or day by day researchers' activity. [1]

Modern scientific models are nothing else but simple representations of complex objects, systems or events and all of these models are used as tools for understanding the nature, the population, the entire world and sometimes even universe. Models use familiar words, notions, objects to represent unfamiliar situations, events, things. Modelling is that kind of action which can help scientists to communicate their ideas, and to understand not only each other but also the processes and phenomena, helping all to make predictions. A modern model is indeed a simplified image that approximates the real complex world, but allows researchers to easily understand some of the major issues or problems and offer clarity, insight, and hopefully predictive behavior. Models are constructed from familiar objects to represent unfamiliar things. Models can help a researcher to visualize most everything, or to design impossible things in your mind, something that is really difficult to see or understand. The model essence is in its state of equilibrium between necessity and utility. A scientific model, even one empirically tested, can make use of mathematics as language, but that is not strictly necessary, just useful. All the scientific models should have the next basic features: i) all initial assumptions (hypotheses) must be scientifically sound; ii) the model's mathematical language and treatment must be self-consistent values; iii) any model must describe the largest set of the available experimental data.

3. MODELLING AND MODELS' SPECIFICITY IN MODERN SCIENCE

A scientific modelling or some realized scientific models are just simplifications that approximate the real world, but allows one researcher to easily think about problems of simplexity (as simplifying the complexity) of the same reality and get clarity, insight, and hopefully predictive behavior. So modelling and models help people in better understanding. A young researcher can learn better based on visualization, because he was born and still lives in visualization times. But, the great majority of old researchers or old teachers, and a great part of the common people cannot visualize a scientific model, even if it is an image or a solid model. A classical theory meets the conditions of optimization and adequacy to the specific reality, or the object of study of the discipline, if it satisfies at least the next major requirements (Figure no. 1):

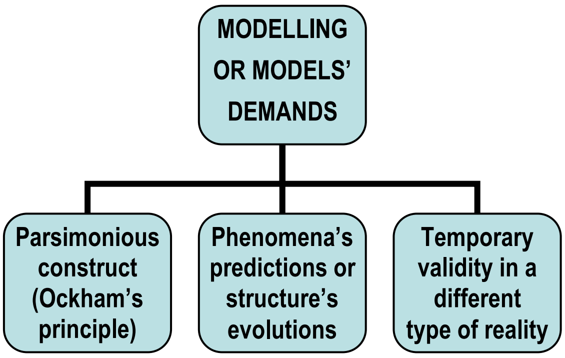

Fig. no. 1. Major Demands of classic modelling

Inter-, multi-, and trans-modelling have new requirements or mandatory needs and all of these can be synthesized as follows (Figure no. 2):

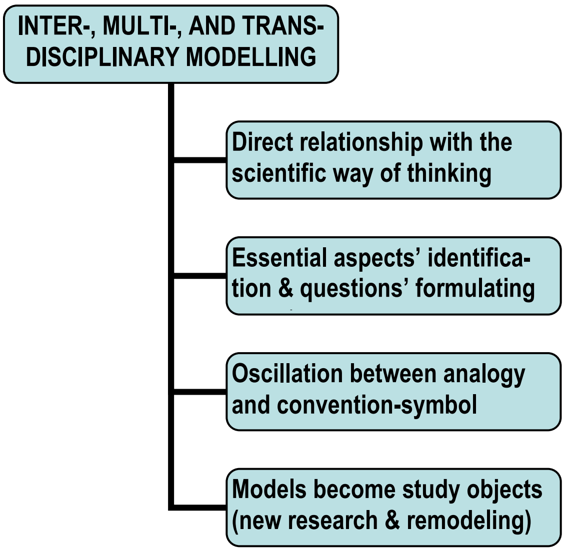

Fig. no. 2. Some essential principles of modeling

The disciplinary multiverse of today's scientific research seems to amplify the requirements of acknowledging and validation of a theory, cyclically considered as superannuated, and permanently perfectible (i.e. a theory can survive only to the extent to which its predictions are ascertained).

The theory of any scientific universe becomes, in the multiverse, a particular case of a theory much vaster in point of applicability, not yet discovered or formulated, while the new theories of the multiverse are inferences, maximized in point of coverage degree and minimized in point of mathematical and logical formulation, of the old theories, extended and selected; this fact is actually acknowledged in the very *principle of complemen-tariness* in physical thought, meaning that the old theories are particular limit cases of the new theories (where the limit, for instance in the theory of general relativity, is the speed of light, and in the theory of quantum physics – Planck's constant).

The final goal of scientific research, or even of science in general, is to provide a unique theory to supply research with a stable support in knowing and anticipating the cosmic multiverse. The multi-disciplinary model turns to account the language and methods of mathematics, testing and statistical decision, the pattern of physics in assessing reality (quantum, thermodynamic, acoustic, etc.), as well as the real variables of the specific subject to research (money flow in the economy, human behaviour in sociology, etc.). The architecture of multidisciplinary modelling capitalizes on shifting from only one science to many sciences or to a multidisciplinary model, through successive (uni)disciplinary models (improvement through imitation, analogy, and passing from one type to another).

Any inter- and multidisciplinary model can be described as an image of a specially selected part of reality, with the aid of which answers can be given to various questions, or problems belonging to an assortment of minimum two domains or fields in the area of scientific knowledge can be solved, with a certain degree of realism and with a certain limit of error. The transdisciplinary model is a result of multiple levels of reality (information theory, scientific modelling theory, systems theory etc.) [4;8]. The sad balance of the predictions made by the econometric models over the past few years, for all the modern calculation equipment added to the sophisticated classical or (uni)disciplinary models, is nothing but an additional confirmation.

All the sciences realistically recognize the impossibility of absolute modelling knowledge, but also any inter-, multi- and transdisciplinary modelling significantly increases the degree of knowledge, anticipation, structuring, etc. of that investigated reality. Emil du Bois - Reymond's famous statement "ignoramus et ignorabis" *(we don't know and we won't know ... everything* - n.a.) continuously contains a grain of truth, be it pure or only relatively. To a truth closer or farther from purity, more or less relative, (uni)disciplinary or multidisciplinary revealed, evolving from inter, to multi- or transdisciplinary, respectively one can formulate some major principles of inter-, multi- and transdisciplinary modelling and for researchers' predictions and simulations, based on these kind of interesting models [9] but, especially, paradoxically expressed. An expanded list of the major principles of inter-, multi- and transdisciplinary modelling and models must contain:

"*1. The harmony of modelling disagreements is a concord of discordances.*

2. The developmental cycle is the axis of the cyclical development.

3. The motion through an apparent state of rest, and the state of rest of the motion are the realities of all the cases of modelling. As a paraphrase to one of Schlozer's dictums, science remains history at rest, very much as history becomes science in motion.

4.The identification of the leap, or the unpredictable transformation, in the sense of the paradox of the arrow, or of the tortoise which overtakes Achilles, represents the spirit of modelling.

5. Communication, as an aim of getting out of information isolation, constitutes the message of modelling.

6. The relativity of the global interdependencies and of the local ones derives from the logic of the systems modelled, namely when the sum of the parts is greater than the whole.

7. The infinite, as part of the finite, and the finite as part of the infinite, describe the structures of modelling.

8. The finality of the inductive through deduction, and the validation of the deductive through induction bound the reasoning of those who do the modelling.

9. Knowledge is the limit to the ignorance of the act of modelling, no less than ignorance eventually becomes the result of knowledge.

10. The rebirth of theory through experiment brings about the demise of experiment in modelling.

11. The faith in critical science becomes similar to the neutrality of ignorance in the acts of modelling.

12. Coherent superposition brings together the amplitudes as limits, while incoherent superposition unites only the intensities through modelling.

13. Finding nuances is a solution of probabilistic thought, and based on the possibilities of modelling.

14. Convergence through divergence contributes to the emergence of modelling.
15. The incompleteness of completeness adds to the completeness of incompleteness in modelling.
16. The compensation of the reactions confers equilibrium to imbalance.
17. The duality of the acts of modelling is a reflex of the equivalence causes-effects.
18. A fixed multidisciplinary modelling method is no method.“[6]
19. “*A model contains its own non-model, within itself or in its essence.*
20.The science of economics (financial economics) is nothing more than a long succession of econometric (financial econometric) models.” [1]

All models are the expressions of some systemic approaches based on the principles of systems theory, from the principle of procedural and structural hierarchy, to the principle of dualism (dichotomy, dissonance), the principle of conservation of substance and energy, the principle of variation (general motion, oscillation, cyclicity, randomness and relativity), the principle of reactive delay or inertia, the principle of threshold value, tolerance, critical quantity, sensitivity, up to the principle of interaction.

In the case of complex real systems (political, economic, social, demographic, ecological, etc.) the modelling becomes irreplaceable, presenting two great advantages: a) pure representation of the phenomenon, process, object subject to research, without being distorted by foreign phenomena or superfluous details; b) performing experiments or performing scenarios, where this activity would be impossible due to the inaccessibility of the real object or the high cost of real action. The preservation of models or their abandonment is dictated mainly by the quality of the predictions, estimates and simulations that capitalize on them.

4. SOME FINAL REMARKS

The scientists doing modelling all day long or who work with the models the entire life will probably develop intuition. All the types of inter-, multi-, and transdisciplinary models can develop a special intuitive understanding of a system, and a good talent for estimations in a variety of normal or abnormal circumstances. An important kind of intuition comes from experience, coming from simplifying subsystems to their essential subsystems, factors, variables, structures etc., Another invaluable type of intuition is coming from accurate measurements, friendly learning instruments, simple system's governing equations, and especially from predictions, and testing all predictions. A complex model, made from hundreds or thousands of equations, variables and interactions between all the variables becomes an opportunity for a better intuition.

A memorable inter-, multi-, and transdisciplinary model must have a memorable name, a simple design, a useful algorithm to solve the real problems, a precise description of phenomenon that makes testable vision or foresight. Starting from a statistical and logical methodology, a memorable inter-, multi-, and transdisciplinary model must be also a functional instrument created to improve some explanations, to promote discussions, to make forecasts, predictions or anticipations, to offer visual images of abstract concepts etc.

There are some modelling paradoxes, coming also from a good intuition of modelling process:
1. “*Model never “proves” in the common sense.*
2. *Most of the models are wrong, but if one researcher is really lucky, he or she can find or discover a useful model.*
3. *Some models work so well that it seems silly to regard them as having no connection to reality and more than sure these models are “proved” in a weak sense*.” [10]
4. One researcher can create a “model”, only for manipulating it to get the needed results.
5. A model is like two edged swords: if it is properly used, it can be a boon to the mankind, but in the hands of mad or bad men, it becomes a disaster in the entire world.
6. If one researcher does not get something logical from his model, then he will term it as useless model, in spite of his useless data, structure, algorithms, variables …
7. Another model paradox is its own state of equilibrium between necessity and utility. A scientific model even an empirically tested one, can make use of mathematical language, but that is not strictly necessary, just useful.
8. Science is a systematic process of studying and understanding reality and research is also a systematic investigation, another process of experimenting to establish facts and data. The common differences between science and research are in facts, truths and errors. This aspect creates “the facts, truth and errors paradox of modelling”. A model can explain facts, without finding neither the truths and nor the level of errors,
9. A model can be a substitute of reality, but it cannot be what reality really represents.
10. When model’s set of assumptions or hypotheses solve two or more problems the final theory of modelling can be the result of a lucky coincidence. But when two different models make the same predictions, one researcher must think of finding a significant part of the scientific truth. This is the paradox of believing too much in coincidence (set of assumptions or hypotheses) instead of producing the same predictions.
11. Previous models have been falsified and modern science always replaced all by a new one but replaceable.
12. Always, there is a new inter-, multi-, and transdisciplinary model’s paradigm that rejects the old or classic theory of (uni)disciplinary model’s paradigm. There is a necessary paradigm shift.
13.The new paradox of data’s simplexity is more and more important. Essential attributes of a model are coming from the observed data and from retrieval data. The more data coming from observed facts a model encapsulates, the better it is (complexity), but also more data retrieval (for usage), the more efficiently it retrieves it the better the model (simplicity).
14. The model’s outputs are influenced by the presence of the researchers as observers.
15. Double liar as model’s paradox is a variant of Jourdain's paradox about the opposite sides of a card. In this version of the famous paradox, any model has two opposite sides and the following words are written on

these two opposite sides of a model: A) back side – "*the sentence on the other side of this model is true*"; B) face side – "*the sentence on the other side of this model is false*." [11]

The inter-, multi-, and transdisciplinary models are the future of all modelling actions, and these modern models mean many different levels of knowledge, distinct research, specific education, another correlation between theory, practice, and technology, including morality and ethics to protect communities. Finally, one researcher can separate inter-, multi- and transdisciplinary models, putting all apart from (uni)disciplinary model by impact on prospects or foresight. Multidisciplinarity makes it easier to get better outputs. Interdisciplinarity makes the same thing, more detailed in a specific area, but relative harder than multidisciplinarity.

Transdisciplinarity gets model out of the present reality, and so the model sit around outside not in exile, but in the immediate future.

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